to be gained in converting reference frames, since the assumption of a constant *diffusivity matrix implies* small concentration changes and consequently small differences between reference velocities.

It is noted that the essential difference between the D_{ij} considered here and those given by Curtiss and Hirschfelder (*2*) [also in Bird et al. (*1*)] in the mass reference frame is not due to the difference in reference frames but rather is due to the fact that the latter are defined in terms of all the gradients including the dependent one, with the diagonal elements chosen to be zero. This representation is less useful in

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A Summary of Distributional Effects in Two-Phase Slip Models

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Slip between the phases of a two-phase stream, expressed as the ratio of average phase velocities, is an important variable affecting both operating characteristics and stability limits of such systems. Consequently, a number of models based upon different physical mechanisms have been proposed for predicting its magnitude. One explanation, first proposed by Bankoff (*1*), for at least part of the slip is that the gas phase migrates to the center of the stream where the resistance to flow and the stream velocity gradient are least. Thus, when the phase velocities are averaged over the cross section of the stream, the average magnitude of the gas velocity is greater than that of the liquid due to the distribution of the phases alone. Following Bankoff, this distributional effect has been included in several more recent models (*2, 3, 4, 6, 7*), although it is not always apparent from the analysis at which point the effect enters.

The purpose of this communication is twofold: First, to derive the distribution parameter proposed by Bankoff in a more general way, thus giving it a more general significance, and second, to show in which way this same distributional parameter or a modification of it enters the models presented in references (*2*), (*3*), (*4*), (*6*), and (*7*). However, before proceeding to our objective, it is necessary to define briefly some of the variables of a two-phase stream flowing in a tube or channel.

The cross section of a two-phase stream may be considered a probability field, at each point of which either of two mutually exclusive events may occur: either gas is present, or liquid is present. If gas is present its velocity will be U_g, and if liquid is present its velocity will be U_w. Denote the probability that gas is observed by p. The average magnitude of the gas velocity in this field (the cross-sectional average gas velocity) is

$$\overline{U}_g = \frac{\frac{1}{A}\int_A p\,U_g\,dA}{\frac{1}{A}\int_A p\,dA} \tag{1}$$

A similar averaging procedure gives the cross-sectional average liquid velocity. The local expectation stream velocity, $U = (1-p)\,U_w + p\,U_g$, is averaged over the flow area as follows

$$\frac{1}{A}\int_A U\,dA = \frac{1}{A}\int_A (1-p)\,U_w\,dA + \frac{1}{A}\int_A p\,U_g\,dA \tag{2}$$

Multiplying and dividing the first term on the right-hand side by $\frac{1}{A}\int_A (1-p)\,dA$ and the second term by $\frac{1}{A}\int_A p\,dA$ gives

$$\overline{U} = (1-\alpha)\,\overline{U}_w + \alpha\,\overline{U}_g \tag{3}$$

where α is the fraction of the cross section occupied by the gas phase (void fraction) and is related to the probability p by

$$\alpha = \frac{1}{A}\int_A p\,dA \tag{4}$$

With this beginning, a general expression for Bankoff's distribution parameter can be derived.

Bankoff assumed that at each point of a two-phase stream the velocities of the phases are equal ($U_g = U_w$). He then introduced the profiles of the stream velocity and the probability as power law functions of the distance from the tube wall and integrated over the tube cross section to obtain the slip ratio relationship

$$\frac{\overline{U}_g}{\overline{U}_w} = \left[\frac{1-\alpha}{K-\alpha}\right] \tag{5}$$

where K is a distributional parameter, which is a function only of the exponents of the power law profile relations.

Bankoff's assumption can be written as

$$U_g = U \tag{6}$$

and integrated over the flow area as done in Equation (1) to obtain the average gas velocity

$$\overline{U}_g = \frac{\frac{1}{A}\int_A p\,U_g\,dA}{\frac{1}{A}\int_A p\,dA} = \frac{\frac{1}{A}\int_A p\,U\,dA}{\frac{1}{A}\int_A p\,dA} \tag{7}$$

which, if we define

$$\frac{1}{K'} = \frac{\frac{1}{A}\int p\,U\,dA}{\left[\frac{1}{A}\int p\,dA\right]\left[\frac{1}{A}\int U\,dA\right]} \tag{8}$$

gives

$$\overline{U}_g = \frac{1}{K'}\,\overline{U} \tag{9}$$

Substitute Equation (3) into the right-hand side of Equation (9) and after

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INFORMATION RETRIEVAL*

Unsteady state mass transfer from gas bubbles-liquid phase resistance, Li, Pang-Sheng, Frank B. West, William H. Vance, and Ralph W. Moulton, **A.I.Ch.E. Journal, 11,** No. 4, p. 581 (July, 1965).

Kew Words: A. Mass Transfer-8, 7, Mass Transfer Coefficients-8, Distillation-8, Absorption-8, Liquids-9, Gases-9, Bubbles-9, Oxygen-1, Carbon Dioxide-1, Chlorine-1, Unsteady State-0, Water-5, Rates-6, Shape-6, Velocity-6, Hydrolysis-6, Reaction-6, Irreversible-0, Kinetics-8, Penetration Theory-8.

Abstract: A well established theoretical basis for mass transfer from gas bubbles is important for predicting distillation and absorption efficiencies. Liquid phase mass transfer coefficients were determined for streams of bubbles in water. The gases used were pure oxygen, 10% carbon dioxide in air, and 10% chlorine in air.

Analytical and numerical solutions for laminar flow of the non-Newtonian Ellis fluid, Matsuhisa, Seikichi, and R. Byron Bird, **A.I.Ch.E. Journal, 11,** No. 4, p. 588 (July, 1965).

Key Words: A. Flow-8, Isothermal-0, Plates-9, Flat-0, Tubes-9, Films-9, Annuli-9, Fluids-9, Non-Newtonian-0, Ellis Fluid-9, Ellis Model-8, Ellis Number-8, Solutions-9, Polymers-9, Distribution-8, Velocity-9, Pressure-9, Shear Stress-9, Rheology-8. B. Flow-8, Tubes-9, Nonisothermal-0, Fluids-9, Non-Newtonian-0, Ellis Fluid-9, Ellis Model-8, Ellis Number-8, Temperature Profile-8, Nusselt Number-8, Heat Transfer-8, Rheology-8.

Abstract: A summary is given of analytical and numerical solutions for the three-constant Ellis model. The solutions include isothermal flow between flat plates in tubes, in films, in annuli (axial, tangential, and radial), and nonisothermal flow in tubes with constant wall temperature and with constant heat flux at the tube wall.

Flash X-ray analysis of fluidized beds, Romero, Jacob B., and Don W. Smith, **A.I.Ch.E. Journal, 11,** No. 4, p. 595 (July, 1965).

Key Words: A. Analysis-8, Fluidized Beds-9, Flash X-Ray Radiography-10, Measurement-8, Density-9, Voids-9, Velocity-9, Air-5, Sand-5, Structure-9, Two-Phase Flow Theory-8.

Abstract: Flash X-ray radiography was used to study the internal structure of fluidized beds. Basic data were obtained on the density distribution and on void shapes, sizes, and velocities within an air-sand bed. The data obtained, in general, support the view of fluidized beds as consisting of liquidlike emulsions through which voids rise. Density measurements and void characteristics were in agreement with two-phase flow theory and with more recent theories of fluid bed flow. Void velocities and shapes agreed with recent predictions that the dense phase behaves as a liquidlike emulsion of zero viscosity and surface tension.

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rearranging Equation (5) is obtained, with $K' = K$.

This derivation shows that, in a general sense, the distribution parameter is a spatial correlation factor between the probability of gas existing at a point in the flow and the stream velocity at the same point. It includes the power law profiles as a special case but can also be evaluated for other profile forms.

The variable density model was modified (*2*) to treat the case when local slip is not small by replacing the assumption that the local phase velocities are equal by an assumption that they are proportional. Thus

$$U_s = c\,U_w \tag{10}$$

The stream velocity can then be written as

$$U = \left[p + \frac{1-p}{c}\right] U_s \tag{11}$$

which is averaged over the flow cross section to give

$$c\overline{U} = (c-1)\,\alpha\,U_s + \frac{1}{A}\int_A U_s\,dA \tag{12}$$

A modified distribution parameter is defined as

$$K_M = \frac{\alpha \int_A U_s\,dA}{\int_A p\,U_s\,dA} = \frac{\frac{1}{A}\int_A p\,dA\,\frac{1}{A}\int_A U_s\,dA}{\frac{1}{A}\int_A p\,U_s\,dA} \tag{13}$$

which allows Equation (12) to be written after rearranging as

$$\frac{\overline{U_s}}{\overline{U_w}} = c\left[\frac{1-\alpha}{K_M-\alpha}\right] \tag{14}$$

These two distribution parameters defined by Equations (8) and (13) are closely related, since K_M includes K as the special case of $c = 1$.

Levy (*3*) presented a two-phase flow model based upon mixing length theory, and gave equations and graphs from which the distributions of density and velocity can be obtained. He did not derive an explicit equation giving the slip ratio. However, an assumption was made, fundamental to the analysis, from which an equation can be derived to show the relation of the model to the distribution parameter derived here. He assumed that the local gas velocity is proportional to the local stream velocity. Thus

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* For details on the use of these Key Words and the A.I.Ch.E. Information Retrieval Program, see **Chem. Eng. Progr.,** Vol. 60, No. 8, p. 88 (August, 1964). A free copy of this article may be obtained by sending a post card, with the words "Key Word Article" and your name and address (please print) to Publications Department, A.I.Ch.E., 345 East 47 St., N.Y. N.Y., 10017. Price quotations for volume quantities on request. Free tear sheets of the information retrieval entries in this issue may be obtained by writing to the New York office.

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$$U_s = k\,U \qquad (15)$$

where k is a proportionality constant greater than unity which accounts for local slip. Equation (15) is integrated over the flow cross section as in Equation (1), using the distribution parameter as defined by Equation (8), to give

$$\frac{\overline{U}_s}{\overline{U}_w} = \left[\frac{1-\alpha}{\dfrac{K}{k}-\alpha}\right] \qquad (16)$$

This author (*4*) analyzed slug flow where the local gas velocity has two components; it is rising through the liquid with a velocity given by the Taylor relationship (*5*) and, at the same time, it is being carried along at the stream velocity. Thus

$$U_s = U + C\sqrt{gD} \qquad (17)$$

where g is the acceleration due to gravity and D is the diameter of the enclosing channel. Again, integrating Equation (17) over the cross section of the stream as in Equation (1), using Equation (8) and noting that the buoyant component of the slug velocity is constant, we obtain

$$\overline{U}_s = \frac{1}{K}\,\overline{U} + C\sqrt{gD} \qquad (18)$$

Using Equation (3) in Equation (18) gives

$$\frac{\overline{U}_s}{\overline{U}_w} = \left[\frac{1-\alpha}{K-\alpha}\right]\left[1 + KC\frac{\sqrt{gD}}{<U_w>}\right] \qquad (19)$$

where $<U_w> = (1-\alpha)\ \overline{U}_w$ is the superficial liquid velocity.

Equation (18) was used previously, although not derived analytically, by Nicklin et al. (*6*), to correlate results of an experimental slug flow study.

Zuber (7) made a more general derivation than that of reference (*4*) by defining a local drift velocity V as

$$V = U_s - U \qquad (20)$$

which is again integrated to obtain the cross-sectional average gas velocity

$$\overline{U}_s = \frac{1}{K}\,\overline{U} + \frac{\displaystyle\int_A pV\,dA}{\displaystyle\int_A p\,dA} \qquad (21)$$

For slug flow, V is equal to the Taylor rise velocity and Equation (18) is valid. For the bubble flow regimes, Zuber gave other rise velocity rela-

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locus methods, and frequency-response methods. The development is a standard one and, except for some of the examples, is not too different from that found in many electrical engineering texts. The next quarter of the book, however, does apply this linear theory to more complex systems which are of direct interest to the chemical engineer.

The final quarter of the book discusses such nonlinear techniques as phase-plane analysis and the describing-function method and then moves to analog simulation techniques as applied to control systems. This portion of the book is the most interesting; it is unfortunate that a greater percentage of the material covered in the entire book was not devoted to these nonlinear systems.

In general, the book is well written and the examples are clearly detailed. The main drawback is the large amount of linear analysis, but for an undergraduate text this may prove quite valuable.

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Industrial Wastewater Control, C. Fred Gurnham, Editor, Academic Press, New York (1965), 476 pages, $16.00.

Control of water pollution is a subject of high current interest. For several years newspapers and popular magazines have been recounting the ravages of uncontrolled discharge of domestic sewage and industrial wastes into surface water; and public demands for abatement of pollution have been reflected in a great deal of legislative activity at both state and federal levels. Publication of a text and reference work on wastewater control is therefore quite timely.

The editor's introduction defines the various parameters of pollution control and briefly describes the common types of wastewater treatment and management. Each of the contributing authors followed a definite outline consisting of a broad review of his industry, a description of the characteristics and volume of the resulting waste water, a discussion of wastewater treatment processes, and a projection of trends in water quality management. This procedure avoided all needless repetition.

Each of the twenty-four chapters has a brief preface that refers the reader to related material in other chapters. The chapters, which are arranged in logical groups, are: Meat; Fish and Fish Products; Poultry and Eggs; Canned Foods; Starch and Starch Products; Sugar; Fermentation Products; Coal Mining; Metal Mining; Industrial Mineral Mining; Coke and Gas; Iron and Steel; Nonferrous Metals; Petroleum; Inorganic Chemicals; Organic Chemicals; Metal Finishing Products; Pulp and Paper; Textiles; Leather; Power; Atomic Energy; and Transportation.

The authors have assembled a surprisingly large amount of numerical data that are frequently difficult to locate. Thus this is truly a reference work that will be useful equally to the professional consultant and to the plant manager seeking a solution to his own wastewater problem.

There is a single thread that runs through all the chapters: the emphasis on good housekeeping practices and process changes to reduce or eliminate industrial wastes at their sources. These common-sense expedients are sometimes difficult to achieve in old industrial plants, but new installations have provision for wastewater control built into the design.

This book illustrates the great progress industry has made in pollution abatement, but it shows clearly that much more needs to be accomplished and indicates the direction future developments should take. On the other hand, the complex technical and economic problems that must be overcome are well documented.

The literary style of the contributors is attractive and quite uniform, indicating conscientious editorial supervision. There is a good index.

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ERRATUM

In the papers "Vapor-Liquid Equilibria in Hydrogen-Benzene and Hydrogen-Cyclohexane Mixtures" by Richard E. Thompson and Wayne C. Edmister and "Calorimetric Determination of the Isothermal Pressure Effect on the Enthalpy of the Propane-Benzene System" by Lyman Yarborough and Wayne C. Edmister (Vol. 11, No. 3, pp. 457-461 and 492-497, respectively), the captions for Figures 1 and 2 were interchanged. The caption for Figure 1 on page 457 should be interchanged with that for Figure 1 on page 492 and the caption for Figure 2 on page 458 should be interchanged with that for Figure 2 on page 493.

INFORMATION RETRIEVAL

(Continued from page 756)

Unsteady state gas-liquid slug flow through vertical pipe, Street, James R., and M. Rasin Tek, **A.I.Ch.E. Journal, 11,** No. 4, p. 601 (July, 1965).

Key Words: A. Flow-8, Two-Phase-0, Gases-9, Liquids-9, Pipes-9, Vertical-0, Slug Flow-8, Unsteady State-0, Prediction-8, Pressure Drop-9, Mass Balances-10, 8, Momentum Balances-10, 8. B. Slug Flow-8, Velocity-7, Rise-9, Bubbles-9, Length-7, Gases-9, Slugs-9, Liquid-9, Diameter-6, Pipe-9, Flow Rate-6, Pressure-6, Time-6, Pressure Drop-7, Air-9, Water-9, Experimental-0.

Abstract: Macroscopic mass and momentum balances have been used to predict the unsteady state behavior of gas-liquid slug flow through vertical pipe. Equations have been developed to predict the time-dependent pressure drop along the pipe as well as the time-averaged pressure drop. The comparisons between predicted and measured pressure drops, both time-dependent and time-averaged, are satisfactory and appear to support the theoretical model.

Unsteady heat transfer to slug flows: effect of axial conduction, Chu, S. C., and S. G. Bankoff, **A.I.Ch.E. Journal, 11,** No. 4, p. 607 (July, 1965).

Key Words: A. Thermal Conductivity-6, Axial-0, Heat Transfer-7, 8, Unsteady-0, Slug Flow-9, Peclet Number-6, Temperature Profile-7, Integral Transforms-10, Thermodynamic Properties-6, Physical Properties-6, Properties (Characteristics)-6.

Abstract: Solutions are obtained for three illustrative cases of unsteady heat transfer to slug flow, taking axial conduction into account. The magnitude of the correction to the solutions neglecting this effect is shown to be quite appreciable near the leading edge. In agreement with previous steady state estimates, the axial conduction correction becomes negligible for Peclet numbers in excess of 100.

The self-diffusivity of substances in the gaseous and liquid states, Mathur, G. P., and George Thodos, **A.I.Ch.E. Journal, 11,** No. 4, p. 613 (July, 1965).

Key Words: A. Self-Diffusivity-8, 7, 9, Diffusivity-8, 7, 9, Gases-9, Liquids-9, Temperature-6, Correlation-8, Pressure-6, Equations-10, Dimensional Analysis-10, Transport Properties-8.

Abstract: Relationships applicable for the prediction of self-diffusivities of gases and liquids have been developed from available experimental values and an application of dimensional analysis. Separate relationships were found to apply for gases at normal pressures and for gases at elevated pressures. For the liquid state a different dependence on the conditions of temperature and pressure was observed.

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tionships that can be introduced into Equation (21) to obtain a slip ratio equation.

This analysis has shown that the distribution parameter proposed by Bankoff is a general concept applicable to any profile forms and that the slip velocity models referred to here all include this parameter, either directly or indirectly.

NOTATION

A = cross-sectional flow area
c = local slip velocity ratio defined by Equation (10)
C = constant in Taylors slug velocity relationship [reference (5)]
D = diameter
g = acceleration due to gravity
k = proportionality constant between gas velocity and the expected velocity proposed in reference (3)
K = the distribution parameter [Equation (8)]
K_M = the modified distribution parameter [Equation (13)]
p = probability of gas existing at a point in the two-phase stream
U = local expectation stream velocity
U_g = local gas phase velocity
U_w = local liquid phase velocity
V = local drift velocity [Equation (20)]
α = the fraction of the flow cross section occupied by gas (void fraction)
$-$ = (superscript bar) refers to a flow cross-sectional average

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